**Celestial Mechanics A**
Miscellaneous Problems

1. A comet, whose mass m is negligible compared with that of the Sun, is moving around the Sun in a parabolic orbit. When at an end of its latus rectum it collides with and coalesces with a stationary object. The combined object subsequently moves in an elliptic orbit of eccentricity 0.8.

(a)   What was the mass of the stationary object?

(b)   Show that, regardless of the mass of the initially stationary object, the eccentricity of the orbit of the combined mass cannot be less than $1/\sqrt{2} = 0.707$.   I.e. the orbit cannot be more circular than in the drawing below.

$$e = 1/\sqrt{2}$$
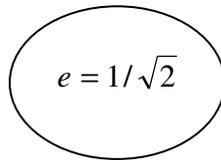
[You may well object that the comet is unlikely to encounter a mystery object that is somehow suspended stationary in space just waiting for the comet to collide with it. Your objections are well founded, but don't use that as an excuse for not doing the problem! You may have to imagine that The Almighty is holding the object lightly between Thumb and Forefinger, and will let go as soon as the comet strikes it.]

2. Future transportation between cities may be by suborbital flight in an ICBM (InterContinental Ballistic Missile), whose former use, we hope, will belong to the distant past.

Assume that Earth is perfectly spherical, is not rotating, and has no atmosphere!   It mass is M, its radius is R, and the universal gravitational constant is G.

If you want to travel by ICBM between two cities separated by an angle 2θ, and you want to travel in a trajectory of least energy, what should be the launch velocity (i.e. speed V_0 and direction α above the horizontal)?

If $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $R = 6371$ km, $M = 5.972 \times 10^{24}$ kg, and $2\theta = 50°$, how long does the journey take?

2θ
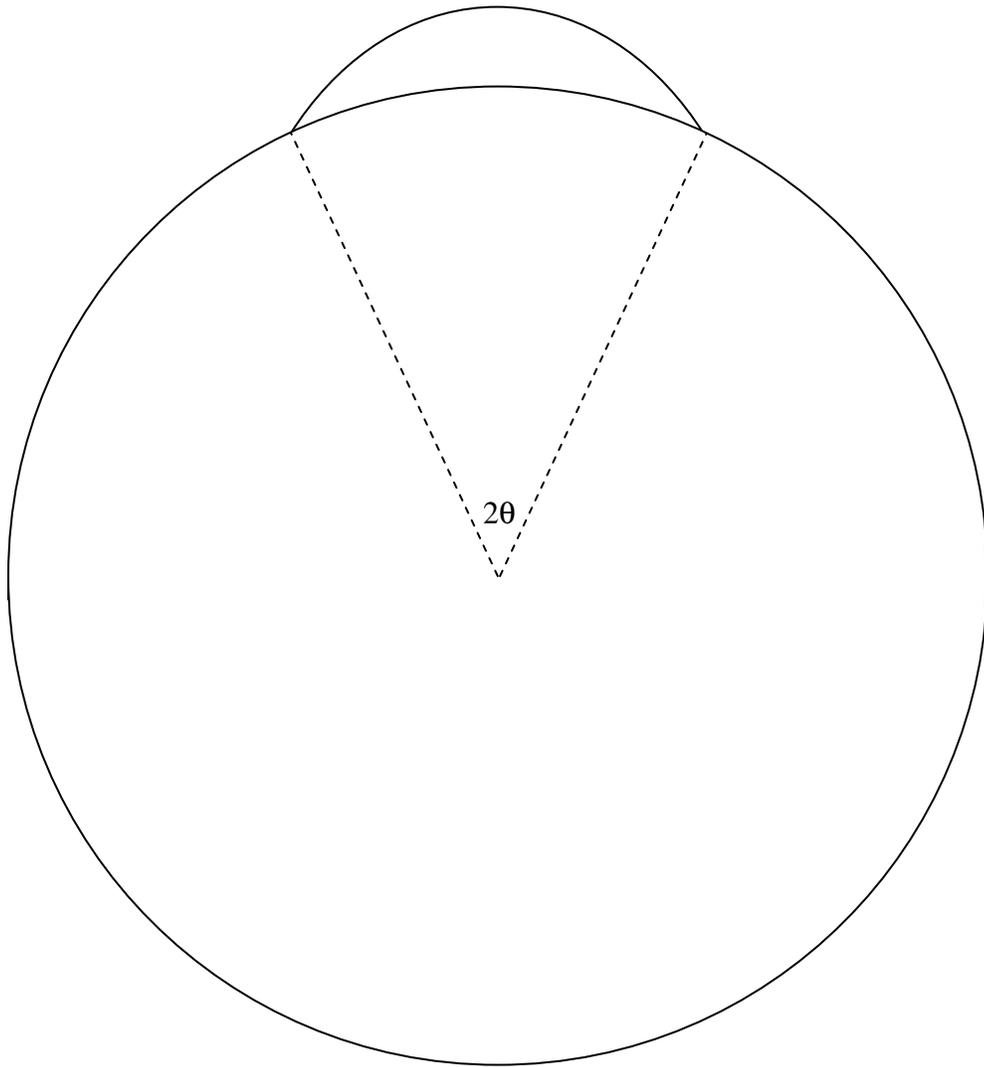
3.
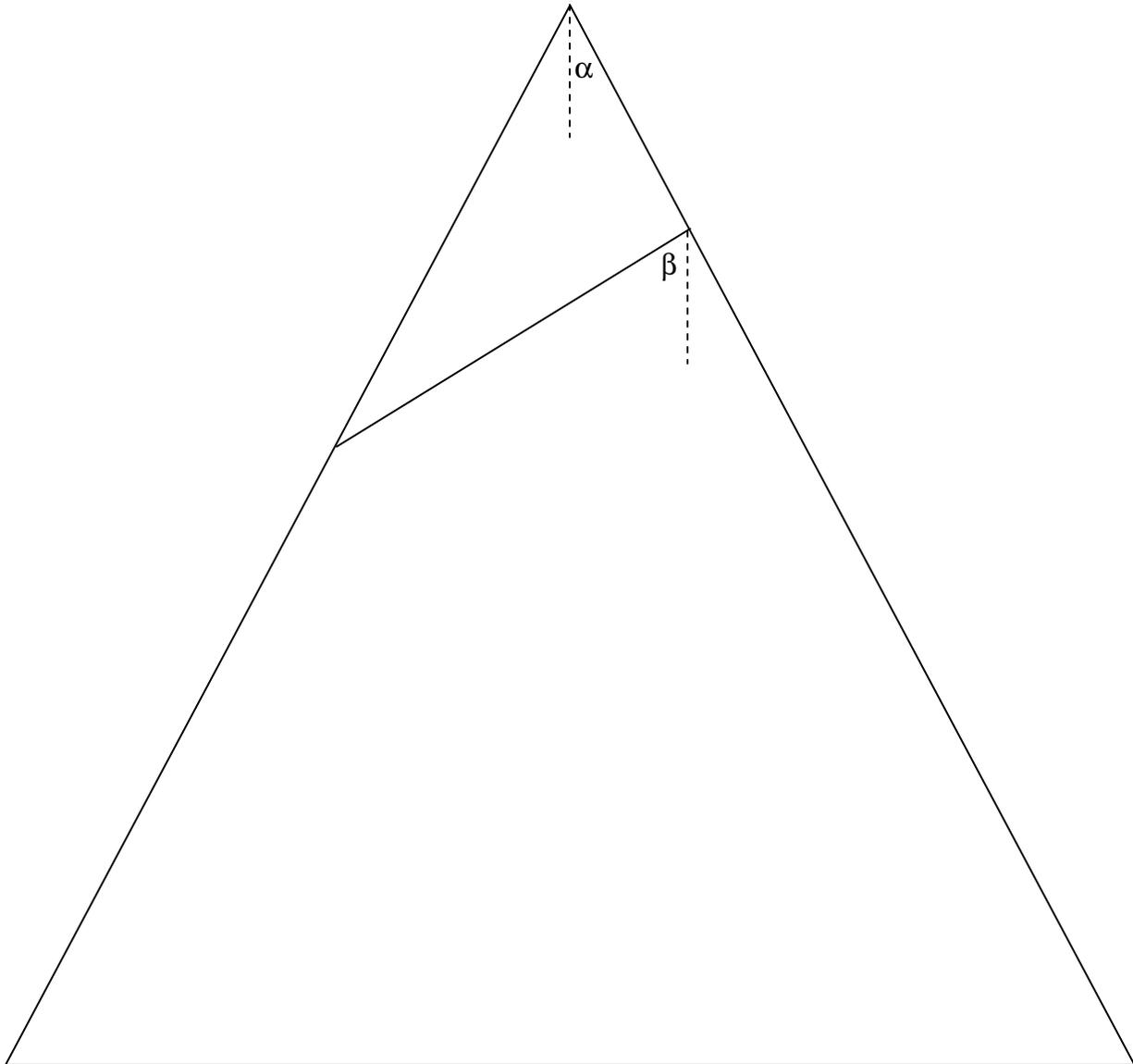
A plane that makes an angle β with the vertical cuts a cone of semivertical angle α. What is the eccentricity of the resulting conic section?

4. An asteroid, whose mass $2m$ is negligible compared with that of the Sun, is moving around the Sun in an elliptic orbit of semi major axis a. When at a distance r $(> \frac{5}{4}a)$ from the Sun it experiences an explosion, which splits it into two equal parts and doubles the kinetic energy of the system. (The perceptive will understand, of course, that there are no external torques on the system, so the angular momentum of the system is

unchanged by the explosion. But there was no need for me to tell you that.) One of the halves pursues a circular orbit, and the other pursues an elliptic orbit. What is the semi major axis of the latter?